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Book Review: *Quantum Inverse Scattering Method and Correlation Functions*

Quantum Inverse Scattering Method and Correlation Functions. V. E. Korepin, N. M. Bogoliubov, and A. G. Izergin, Cambridge University Press, Cambridge, 1996.

A central problem in quantum many-body theory is the description of the long-time dynamics of strongly interacting systems at non-zero temperatures. The basic physical phenomena that are believed to occur in the long-time limit are those of relaxation and transport. Because of the non-zero temperature, the system is clearly not in its ground state, and any local fluctuations are expected to relax back to the thermal equilibrium ensemble either by quantum tunnelling or thermal activation. Quantities which are conserved have to be transported from point to point, and are characterized by a diffusion constant. Despite their importance and ubiquity our understanding of theses processes is based almost entirely upon quantum kinetic equations (these are generalizations of the Boltzmann equation of a dilute classical gas) which can only be justified in a rather ad-hoc manner for the case of a system with weakly interacting excitations. The Landau theory of the low temperature properties of a Fermi liquid is an example of a very successful theory of this type.

In many areas of modern condensed matter physics, e.g., in the study of the high temperature superconductor, it has become clear that it is urgently necessary to go beyond this weakly interacting paradigm. Many interesting physical phenomena occur at intermediate temperatures and in the presence of strongly interacting excitations, and reliable results for systems of this type quite scarce.

For one class of strongly interacting systems, known broadly as "quantum impurity" problems, much of the non-zero temperature dynamics is now understood. A decade of work, relying on various approximate techniques and physical insights, has been nicely summarized in the book *Dissipative Quantum Mechanics* by G. Weiss (World Scientific, Singapore, 1993). In a recent beautiful series of papers, Hubert Saleur and collaborators have

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extended this work, while formulating it in an elegant and rigorous manner which clearly exposes the universal characteristics of the spectral functions. The simplifying feature of these quantum impurity problems is that all of the strong interactions are localized with a single degree of freedom or at a single point in space.

For more generic quantum many body systems with strong interactions in the bulk, there were essentially no well established results until recently. Beginning in the early 80's, a group of mathematical physicists working mainly at the Steklov Mathematical Institute in St. Petersburg made a concerted attack on the problem of a Bose gas of particles with short-range repulsive interactions in one dimension. They used the existing arsenal of the so-called inverse scattering method, and developed a number of new techniques to understand quantum systems at non-zero temperature. Their work has yielded interesting exact results for many observables including those describing the time-dependant relaxation of order parameter at non-zero temperature.

These impressive and important developments have now been presented in the book *Quantum Inverse Scattering Method and Correlation Functions* by three of the leaders of the effort V. Korepin, N. M. Bogoliubov and A. G. Izergin. The book is a very welcome addition to the literature as it largely succeeds in making these advances accessible to a wider audience. In principle, the book can be read by a physicist with the canonical training in the basic graduate curriculum, and no prior exposure to exactly solvable systems. However, I suspect that even experienced researchers in integrable systems, who are not interested specifically in the dilute Bose gas, will find this a very useful reference, as it collects together much of basic technology in a single coherent package.

I found the introductory chapters of the book the most useful. Chapter 1 contains a clear and elegant discussion of the Bethe ansatz for the eigenstates of the impenetrable Bose gas. I have attempted to understand the ansatz in many other contexts before, but this was the first time I felt I was able to fully penetrate the discussion, and gain an intuitive understanding of what made ansatz work. The subsequent chapters contain further discussion of the Bethe ansatz in a number of other models, but these are plain sailing after having understood the method in its simplest context in Chapter 1.

The inverse scattering method is introduced in Chapter 5, first in the context of classical models. Most readers will probably find this discussion too terse, and will need to consult the introductory chapters of the book *Hamiltonian Methods in the Theory of Solitons* by L. D. Faddeev and L. A. Takhtajan (Springer-Verlag, Berlin 1987) to gain a better intuitive understanding. The subsequent chapters present a synthesis exposing the relationship between important techniques in integrable systems: the Bethe ansatz, the inverse scattering method, and the Yang–Baxter approach to classical statistical mechanics problems. This discussion is quite technical and the chapters require numerous rereadings, but the global understanding gained makes them quite worthwhile.

The remainder of the book now approaches the specific problem of computing non-zero temperature correlations of the Bose gas. The mathematical apparatus required becomes steadily more unwieldy for most readers. A key intermediate step is the determinant representation of these correlators. This is first derived using the inverse scattering method and then using a form factor approach. I wish the presentation had been in the opposite order as the latter method is certainly simpler, and even relatively elementary, for the case of the Bose gas. The determinant representation is then used to derive non-linear differential equations for the correlators, and these turn out to be classical integrable equations of the type studied in Faddeev and Takhtajans's book. The solution of these equations is obtained by the mapping to the Riemann-Hilbert problem (loosely speaking this the problem of factorizing a function of a complex variable into pieces which are analytic in either the upper or lower half planes.) This discussion was well beyond my abilities to comprehend, and really constitutes a very rapid and terse course at the forefront of applied mathematics. At the very least a reading of a substantial part of Faddeev and Takhtajan's book is prerequisite.

It is clear that this book is a required possession in the library of any physicists with an interest in the integrable systems. It is a valuable, albeit quite technical, reference to a half century of important developments in this subject. At the same time, its introductory chapters also offer an entre to the novice researcher into this imposing edifice. For these reasons I suspect it will serve as a springboard for the subsequent progress in understanding the dynamics of strongly interacting quantum systems at nonzero temperature.

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